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CHEAP TALK IN BARGAINING GAMES

By
Joseph Farrell
and
Robert Gibbons*

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massachusetts
institute of
technology

50 memorial drive
cambridge, mass. 02139

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*GTE Labs and Department of Economics, M.I.T., respectively. We thank Peter Cramton, William Samuelson, and the M.I.T. Theory seminar for helpful comments.

ABSTRACT

This paper shows that cheap talk can matter in bargaining. We analyze a two-stage bargaining game in which cheap talk may be followed by serious negotiation. Cheap talk matters because it can affect whether negotiation ensues. The conventional wisdom, that all buyers would claim to have low reservation prices, assumes that participation is determined exogenously, and is incorrect in our model.

One of the equilibria in this game maximizes both the ex-ante expected gains from trade and the ex-ante probability of trade, but does not involve cheap talk in an important way. We focus on a different equilibrium---one in which cheap talk is central. This equilibrium performs less well ex-ante, but once they learn their reservation prices, exactly as many types of each party strictly prefer the cheap-talk equilibrium as strictly prefer the ex-ante efficient equilibrium.

KEYWORDS: Cheap Talk, Communication, Bargaining, Efficient Trading, Mechanism Design, Game Theory.

1. Introduction

One Saturday evening, two corporate moguls have a chance encounter at their country club. One mogul's company owns a division that the other mogul's firm may wish to buy. Serious negotiation, involving binding offers and hordes of lawyers, can take place on Monday morning; all that can happen Saturday night is talk. If, based on this talk, the moguls conclude that there is sufficient prospect of gains from trade, then they will send their lawyers into the fray on Monday morning. Otherwise, Saturday evening will be the end of it.

This paper shows that such talk can matter in bargaining. We analyze the moguls' two-stage bargaining problem in a model that avoids two unrealistic features of much recent work on bargaining under incomplete information. These unrealistic features are: First, the sets of "types" of buyers and sellers who negotiate are exogenous, even though (typically) in equilibrium some of these types never trade. And second, most analyses consider only payoff-relevant choices, such as binding offers that the opponent can accept, or moves that impose costs of delay.

These two assumptions go naturally together. If indeed the participants are exogenously determined, then it is easy to see that payoff-irrelevant communication (cheap talk) can have no effect: every type of buyer would like the seller to believe that his reservation price is low; and conversely every seller wants the buyer to believe that his valuation is high. Thus, costless messages are never credible if participation is exogenous. This is presumably why cheap talk has not hitherto appeared in the bargaining literature.

But if potential buyers and sellers choose whether or not to take part in serious bargaining, so that the set of participants is not exogenous, then

there is a role for cheap talk. People commonly explore mutual interest through completely non-binding, payoff-irrelevant means before undertaking detailed negotiation. Only if there is enough prospect of gains from trade will formal bargaining ensue. In this paper we analyze agents' incentives in this cheap-talk phase, when they must choose how enthusiastic to appear.

For familiar reasons, appearing too keen harms one's bargaining position. Against this, however, seeming too reluctant jeopardizes the continued negotiation and hence risks losing the gains from trade altogether. This tradeoff creates a role for cheap talk. Buyers with high reservation prices are willing to show their eagerness in order to ensure serious negotiation, even at the cost of spoiling their bargaining position. Buyers with low reservation prices are coy: they feign lack of interest in the hope that the seller will cajole them to the bargaining table, where they will enjoy a favorable position.

The rest of the paper makes this intuition precise, first with an example and then with some general results.

2. An Example

We consider a model of bilateral trade under incomplete information. If the parties do meet on Monday, they play the following extensive-form game (following Chatterjee and Samuelson (1983)). Buyer and seller name prices p_b and p_s respectively, and trade takes place at price $(p_b + p_s)/2$ if $p_b > p_s$; otherwise, there is no trade.[1]

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[1]For those who miss the lawyers, consider the commitment necessary to play even this simple game: what, for instance, stops one party from reneging on his offer in order to capitalize on the information conveyed by the other party's offer?

On Saturday, however, the parties can engage in cheap talk. We consider the simplest possible language: each party can claim either to be "keen" or to be "not keen". We also assume that these claims are made simultaneously. We emphasize that these claims do not directly affect payoffs: they work <u>only</u> through affecting the other player's beliefs. In particular, they are not commitments nor are they verifiable.

To summarize, the extensive form is as follows. First, the parties simultaneously announce whether they are "keen" or "not keen"; these announcements do not directly affect either party's payoff. After observing the pair of announcements, the parties simultaneously decide whether to go to the bargaining table. If both parties arrive at the bargaining table, then they play the Chatterjee-Samuelson game described above; otherwise, the game ends and payoffs are zero for both players. If trade takes place at price p in the Chatterjee-Samuelson game, then a buyer with valuation v_b achieves payoff v_b-p and a seller with valuation v_s achieves payoff p-v_s; if trade does not occur then payoffs again are zero.

In this game, as in every cheap-talk game, there is an uncommunicative equilibrium: if cheap talk is taken to be meaningless, then parties are willing to randomize uninformatively over the possible messages. There are also two more interesting equilibria in which cheap talk is meaningful. In one, serious bargaining takes place only if both parties claim to be "keen"; in the other, a single such claim suffices. In both of these equilibria, we take it that serious bargaining cannot occur if neither party claims to be "keen": because of the need to coordinate on when and where to meet on Monday, an attempt to arrange a meeting belies a party's claim that he is "not keen".

In the first of these equilibria with meaningful cheap talk, the Chatterjee-Samuelson equilibrium reappears: everyone claims to be "keen" except those types who are sure not to trade.[2] In this equilibrium, cheap talk is credible, but does not affect the equilibrium outcome: the outcome is the same as in the Chatterjee-Samuelson equilibrium without cheap talk.

In the other equilibrium, however, cheap talk really matters: low-value buyers and high-value sellers are willing to jeopardize continued negotiation so as to improve their bargaining position; those who have more at stake cannot afford this risk. We focus on this equilibrium.

We analyze our equilibrium in the standard case in which v_s and v_b are independently and uniformly distributed on [0,1]. In the appendix, we show that the following strategies are a perfect Bayesian equilibrium. In the cheap-talk phase, buyers above the critical type

$$y = \frac{22 + 12\sqrt{2}}{49} \approx .795$$

say "keen" while those below say "not keen". Sellers below (1-y) say "keen", while those above say "not keen".

If both parties say "not keen" then the negotiation ends. If at least one party says "keen" then the bargaining continues with a (possibly asymmetric) Chatterjee-Samuelson game. If, for instance, the seller says "not keen" and the buyer says "keen" then it becomes common knowledge that the seller's type is above 1-y and the buyer's type is above y, and negotiation proceeds on that basis. Similarly, if the seller says "keen" and

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[2] In the familiar case in which the equilibrium strategies are linear and both buyer's value v_b and seller's value v_s are independently and uniformly distributed on [0,1], all buyers with $v_b > 1/4$ and all sellers with $v_s < 3/4$ claim to be "keen". Strictly, the other types of buyers and sellers, who will not trade, may say anything. But if there are any costs of serious bargaining, then they must say "not keen".

the buyer says "not keen" then it becomes common knowledge that the seller's type is below 1-y and the buyer's type is below y. In both of these cases, we use the linear Chatterjee-Samuelson equilibrium to solve the resulting bargaining game. Finally, if the buyer and the seller both say "keen" then it becomes common knowledge that the seller's type is below 1-y and the buyer's type is above y. In this case, the Chatterjee-Samuelson analysis breaks down; we invoke symmetry to assume that trade occurs with certainty at a price of 1/2.

One weakness of this equilibrium is that there remains a small prospect of trading even when both parties say "not keen", so it is not the case that the parties voluntarily stop negotiating because each is too pessimistic about the other's type: in equilibrium, $y>1-y$, so trade would occur with positive probability if negotiation continued.[3] Moreover, if the time and place of Monday's meeting are given exogenously, then this equilibrium relies on weakly dominated strategies: no type of either party ever suffers a negative payoff in the Chatterjee-Samuelson game, so showing up weakly dominates not doing so. This is not an appealing feature of the equilibrium, but we do not find it unpalatable because (1) if there are any costs of showing up on Monday then staying home is no longer a weakly dominated strategy, and (2) if the time and place of Monday's meeting are not given exogenously then an attempt to arrange these details belies a claim to be "not keen".[4] In any case, the strategies described here are an equilibrium:

[3] We wonder whether this feature would disappear in an equilibrium of a new game with either more rounds of cheap talk or (equivalently) one round with a richer language.

[4] This discussion suggests that cheap talk may not matter when, as in a shop or a bazaar, the seller keeps known hours in a known place and the buyer has little cost of returning to re-open the bargaining.

if Saturday's talk leads the seller to conclude that the buyer will not show up for Monday's bargaining, then it is a best response for the seller not to show up, and vice versa.

Calculation shows that the cheap-talk equilibrium yields buyer-type v_b an interim payoff, evaluated before the cheap-talk phase, of:

$$W_b(v_b) = \begin{cases} 0 & \text{if } v_b \leq \frac{1}{4}y \\ \frac{1}{2}(v_b - \frac{1}{4})^2 & \text{if } \frac{1}{4}y < v_b \leq 1 - \frac{3}{4}y \\ (1-y)(v_b - \frac{1}{2} + \frac{1}{4}y) & \text{if } 1-\frac{3}{4}y < v_b \leq y, \\ \frac{1}{2}(v_b - \frac{1}{4}y)^2 - \frac{1}{2}(\frac{7}{4}y - 1)^2 & \text{if } v_b > y. \end{cases}$$

An immediate consequence is that if $y/4 \approx .199 < v_b \leq 1/4$, then buyer-type v_b is strictly better-off in our cheap-talk equilibrium than in Chatterjee-Samuelson's. In fact, many other types are better-off in our equilibrium than in Chatterjee-Samuelson. Equating our $W_b(v_b)$ to the Chatterjee-Samuelson equivalent $W_b^{CS}(v_b) \equiv \frac{1}{2}(v_b - 1/4)^2$ yields a crossover point in the range $1 - \frac{3}{4}y < v_b < y$, given by the solution to

$$(1-y)(v - \frac{1}{2} + \frac{1}{4}y) = \frac{1}{2}(v - \frac{1}{4})^2,$$

which is approximately equal to .599, and indeed is between $1 - \frac{3}{4}y \approx .404$ and $y \approx .795$. Thus, all buyer-types in (.199, .599), and all seller-types in the analogous interval, are better off with cheap talk. In fact, exactly as many types strictly prefer our equilibrium as strictly prefer Chatterjee-Samuelson.

The pairs (v_b, v_s) who trade in our equilibrium are illustrated in Figure 1, which also shows the corresponding region for Chatterjee-Samuelson.

Calculation shows that the (ex-ante) probability of (v_b, v_s) falling into the trading region for our equilibrium is approximately .244, somewhat less than the corresponding probability (.281) for the Chatterjee-Samuelson equilibrium: our equilibrium involves less trade. Similarly, the ex-ante expected total gains from trade in our equilibrium are .124, less than Chatterjee-Samuelson's figure of .140.

Both of these results are special cases of Myerson and Satterthwaite's (1983) general result that the Chatterjee-Samuelson linear equilibrium maximizes both ex-ante probability of trade and ex-ante gains from trade. Myerson (1983), however, convincingly argues that such ex-ante efficiency is often irrelevant, because there is seldom an opportunity to make binding arrangements ex-ante (that is, before either player knows his "type"). Myerson gives an example of an incentive-compatible mechanism (for the independent, uniform case) in which even high-value sellers (and low-value buyers) trade with positive probability, and therefore are better-off than in the Chatterjee-Samuelson equilibrium. Our cheap-talk equilibrium is in the same spirit, but is derived from an explicit extensive-form game (with no mediator).

3. General Results

The intuition given in the Introduction does not depend on anything as specific as the linear equilibrium in the Chatterjee-Samuelson game. We now formally confirm that the cheap-talk equilibrium just described exists quite generally. To keep things simple, we impose symmetry and assume that types are uniformly distributed on intervals contained in [0,1], but we have no reason to suspect that these assumptions are necessary.

Consider an extensive-form bargaining game and a sequential equilibrium in that game. Keeping the bargaining rules fixed, we will be interested in the interim payoffs to players of various types as these types and the associated type spaces vary. Since bargaining games often have multiple equilibria, varying the type-spaces generates a correspondence from type-spaces to sets of equilibria, and hence to sets of interim payoff functions. We will assume that a selection can be made from this correspondence that has certain reasonable properties, described below.

In what follows, we refer to this selection as the <u>bargaining environment</u>, and make assumptions about the resulting payoff functions rather than about the underlying extensive form and equilibrium. This simplifies the exposition a great deal, for instance by allowing us to skip the tedious caveat that symmetric extensive forms can have asymmetric equilibria. More importantly, focusing on the interim payoff functions emphasizes the main point of the paper: cheap talk works by affecting the players' beliefs about each other's types, and thus (via our selection) indirectly affecting their payoffs.

A necessary conditon for our cheap-talk equilibrium to exist is that the buyer-type denoted by y above is indifferent between saying "keen" and saying "not keen". Denote the analogous value of v_s by x. Denote the interim payoff to buyer-type v_b when the types are uniformly distributed on $[\underline{v}_s, \bar{v}_s]$ and $[\underline{v}_b, \bar{v}_b]$ by $U_b(v_b; [\underline{v}_s, \bar{v}_s]), [\underline{v}_b, \bar{v}_b])$, and denote the analogous seller's interim payoff by $U_s(v_s; [\underline{v}_s, \bar{v}_s], [\underline{v}_b, \bar{v}_b])$. Then x and y must satisfy

$$x\ U_b(y;[0,x],[y,1]) + (1-x)U_b(y;[x,1],[y,1])$$
$$= x\ U_b(y;[0,x],[0,y]), \text{ and}$$

$$(1-y)U_s(x;[0,x],[y,1]) + y\,U_s(x;[0,x],[0,y])$$
$$= (1-y)U_s(x;[x,1],[y,1]).$$

We now make several innocuous assumptions on the bargaining environment that guarantee that these indifference conditions have a symmetric solution $y\varepsilon(1/2,1)$ and $x=1-y$, and that a cheap-talk equilibrium of the form we describe exists.

Definition: A bargaining environment with types $v_s \varepsilon [\underline{v}_s, \bar{v}_s]$ and $v_b \varepsilon\ [\underline{v}_b, \bar{v}_b]$ is symmetric if

$$U_b(v;[\underline{v}_s, \bar{v}_s], [\underline{v}_b, \bar{v}_b]) = U_s(1-v;[1-\bar{v}_b, 1-\underline{v}_b],[1-\bar{v}_s, 1-\underline{v}_s])$$

for every $v\varepsilon[\underline{v}_b, \bar{v}_b]$.

Assumptions:

(A1) The bargaining environment is symmetric.

(A2) Trade occurs with positive probability when the type spaces are $\{[\underline{v}, \bar{v}], [\underline{v}, \bar{v}]\}$.

(A3) The seller (buyer) never trades at a price below (above) his type.

(A4) The interim payoff functions are continuous in all their arguments.

(A5) For each $v_b\varepsilon[\underline{v}_b, \bar{v}_b]$, $U_b(v_b;[\underline{v}_s, \bar{v}_s], [\underline{v}_b, \bar{v}_b])$ is monotone decreasing in $\underline{v}_s$, $\bar{v}_s$, $\underline{v}_b$, and $\bar{v}_b$.

Proposition 1: Given (A1)-(A5), there exists an equilibrium in which cheap talk plays the role described above.

Proof: Substituting x=1-y into the indifference conditions and invoking the symmetry assumption (A1) shows that y must solve F(y) = 0, where

$$F(y) \equiv (1-y)U_b(y;[0,1-y],[y,1]) + y\ U_b(y;[1-y,1],[y,1])$$
$$- (1-y)U_b(y;[0,1-y],[0,y]).$$

For y>1/2, the subgame with type spaces $\{[0,1-y],[y,1]\}$ is problematic. As described earlier, we choose the equilibrium in which trade occurs with certainty at a price of 1/2. Hence

$$U_b(y;[0,1-y],[y,1]) = y- \frac{1}{2}.$$

Because of the continuity assumption (A4), a suitable $y\varepsilon(1/2,1)$ exists if $F(1) > 0 > F(1/2)$. This holds because (A2) and (A5) guarantee that $U_b(1;[0,1],[1,1])$ and $U_b(\frac{1}{2};[0, \frac{1}{2}],[0, \frac{1}{2}])$ are positive (because U_b is an increasing function of v_b), and (A3) ensures that $U_b(\frac{1}{2};[\frac{1}{2},1],[\frac{1}{2},1] = 0$.

It remains to check that buyer-types above y prefer to say "keen", and those below prefer to say "not keen". We do this by showing that

$$G(y') \equiv (1-y)U_b(y';[0,1-y],\ [y,1]) + y\ U_b(y';[1-y,1],\ [y,1])$$
$$- (1-y)U_b(y';[0,1-y],\ [0,y])$$

is positive for y'> y and negative for y'< y.

For type spaces I_s and I_b, the derivative of $U_b(v_b; I_s, I_b)$ with respect to v_b is the probability that v_b trades in that bargaining environment. (To see this, apply the envelope theorem: we can assume that when v_b increases slightly, the buyer names the same price p_b.) Now the probability of trade in the first subgame is 1 for all buyer-types above the equilibrium price, which (by symmetry) is 1/2. Therefore the first term dominates the third, and so for $y' > 1/2$, $G(\cdot)$ is increasing. Since $G(y) = 0$ and $y>1/2$, this proves that $G(y') > 0$ for all $y' > y$, and that $G(y') < 0$ for

$1/2 \leq y' \leq y$. When $y' \leq 1/2$, the first term in $G(\cdot)$ vanishes, and the second term also vanishes because of (A5) and the argument that the price is 1/2 in the subgame $\{[0,1-y], [y,1]\}$. Therefore $G(y') \leq 0$ for $y' \leq 1/2$. Q.E.D.

We can also generalize our finding in the Chatterjee-Samuelson example that low-value buyers prefer the cheap-talk equilibrium to the no-cheap-talk equilibrium. Intuitively, this is not surprising: with cheap talk, a low-value buyer can improve his bargaining position by sacrificing the chance to trade with high-value sellers---and this sacrifice is costless or almost costless to him, while imitating the "not keen" message is very costly for high-value buyers.

To give our general result, we must define notation for ex-post payoffs as a function of true types and of beliefs in the bargaining game. Thus, let

$$u_b(v_b, v_s; [\underline{v}_s, \bar{v}_s], [\underline{v}_b, \bar{v}_b])$$

be the ex-post payoff to buyer-type v_b when the true seller-type is v_s and when it is common knowledge that the buyer's beliefs about v_s are uniform on $[\underline{v}_s, \bar{v}_s]$, and that the seller's beliefs about v_b are uniform on $[\underline{v}_b, \bar{v}_b]$. These beliefs might be incorrect, as when the parties are off the equilibrium path. But when they are correct, the expectation of u_b gives the interim payoff U_b:

$$U_b(v_b; [\underline{v}_s, \bar{v}_s], [\underline{v}_b, \bar{v}_b]) \equiv E\{u_b(v_b, v_s; [\underline{v}_s, \bar{v}_s], [\underline{v}_b, \bar{v}_b] \,|\, v_s \varepsilon [\underline{v}_s, \bar{v}_s]\}.$$

To prove the result, we need an extra assumption: given the true seller-type, the buyer becomes better-off if his beliefs about the seller become at once more optimistic and more precise:

Assumption:

(A6) $u_b(v_b, v_s; [\underline{v}_s, \bar{v}_s], [\underline{v}_b, \bar{v}_b])$ decreases monotonically in $\bar{v}_s$ provided

that $v_s \varepsilon [\underline{v}_s, \bar{v}_s]$.

We now have:

Proposition 2: Given (A1), (A3), (A5), and (A6), all buyer-types $v_b \leq 1-y$ (and all seller-types $v_s \geq y$) are better-off in our cheap-talk equilibrium than they are in the same bargaining environment without cheap talk.

Proof: We need to show that, when $v_b \leq 1-y$,

$$U_b(v_b; [0,1], [0,1]) \leq (1-y)\ U_b\ (v_b; [0,1-y],\ [0,y]).$$

We first prove that

$$U_b(v_b; [0,1], [0,1]) \leq (1-y)\ U_b(v_b; [0,1-y], [0,1]).$$

By iterated expectation,

$$\begin{aligned} U_b(v_b; [0,1], [0,1]) &\equiv E\{u_b(v_b,\ v_s; [0,1], [0,1]) \mid v_s \varepsilon [0,1]\} \\ &= (1-y)\ E\{u_b(v_b,\ v_s; [0,1], [0,1]) \mid v_s \varepsilon [0,1-y]\} \\ &\quad + y\ E\{u_b(v_b,\ v_s; [0,1], [0,1]) \mid v_s \varepsilon [1-y,1]\}. \end{aligned}$$

But by hypothesis v_b never trades with seller-types above 1-y, so the second term disappears. Now apply (A6) inside the expectation in the first term: a change in the buyer's beliefs from "$v_s \varepsilon [0,1]$" to "$v_s \varepsilon [0,1-y]$" makes him better-off ex-post for all true types $v_s \varepsilon [0,1-y]$. Therefore $U_b(v_b; [0,1], [0,1])$ is less than or equal to

$$(1-y) E\{u_b(v_b,\ v_s; [0,1-y], [0,1]) \mid v_s \varepsilon [0,1-y]\} = (1-y)\ U_b(v_b; [0,1-y], [0,1]).$$

To conclude the proof, note that

$$U_b(v_b; [0,1-y], [0,1]) \leq U_b(v_b; [0,1-y], [0,y]),$$

because of (A5). Q.E.D.

4. Conclusion

This is in part a polemical piece. We believe that economics and game theory have greatly underestimated the importance of costless, non-verifiable, informal communication. This paper introduces cheap talk to bargaining games, but cheap talk itself is not new. The seminal work, by Crawford and Sobel (1982), shows that cheap talk may be credible if agents' interests are not completely opposed. In bargaining, agents are in conflict over the price if trade occurs, but have common interests in consummating trade when the buyer's value exceeds the seller's.

In general, cheap-talk games have multiple equilibria. Ours is no exception. Unfortunately, standard refinement techniques such as that of Cho and Kreps (1985) have no effect. Farrell (1986a) has taken the first steps towards a refinement technique for cheap-talk games, but it is not yet clear whether this helps in our problem.

Cheap talk can be important in economic settings other than bargaining. Farrell (1986b), for instance, studies cheap talk between potential entrants in a natural monopoly, Farrell and Saloner (1986) consider cheap talk between potential adopters of a new technology, Gibbons (1986) models arbitration as a cheap-talk game, and Sobel (1985) develops a theory of credibility in finitely repeated relationships. The fundamental insight that cheap talk can be credible in variable-sum games, combined with the ubiquity of such talk, suggests that a rich collection of other applications lies ahead.

Appendix

In this appendix we adapt the Chatterjee-Samuelson analysis to suit our purposes, and then use the results to derive the equilibrium value of y.

Chatterjee and Samuelson consider a bargaining game with seller-type v_s uniformly distributed on $[\underline{v}_s, \bar{v}_s]$ and buyer-type v_b independently and uniformly distributed on $[\underline{v}_b, \bar{v}_b]$. Both parties name prices, p_s and p_b, and trade occurs at the average of the two prices if the buyer's price exceeds the seller's.

As Chatterjee-Samuelson show, an essential part of the equilibrium is the solution of a linked pair of differential equations, and one solution (on which we and they focus) is linear:

$$\begin{aligned} \tilde{p}_s(v_s) &= \frac{2}{3} v_s + \frac{1}{4} \bar{v}_b + \frac{1}{12} \underline{v}_s, \text{ and} \\ \tilde{p}_b(v_b) &= \frac{2}{3} v_b + \frac{1}{4} \underline{v}_s + \frac{1}{12} \bar{v}_b. \end{aligned} \tag{1}$$

When these functions imply that no type of either party is sure to trade (that is, $\tilde{p}_b(\bar{v}_b) \leq \tilde{p}_s(\bar{v}_s)$ and $\tilde{p}_s(\underline{v}_s) \geq \tilde{p}_b(\underline{v}_b)$), then the equilibrium strategies are $p_s(v_s) = \tilde{p}_s(v_s)$ and $p_b(v_b) = \tilde{p}_b(v_b)$.

If, on the other hand, these functions make one party sure to trade, then there is an incentive to deviate, and the equilibrium is modified as follows: If some type of some player is not sure to trade, then the buyer-type v_b names the price $p_b(v_b) = \min(\tilde{p}_b(v_b), \tilde{p}_s(\bar{v}_s))$ and the seller-type v_s names the prices $p_s(v_s) = \max(\tilde{p}_s(v_s), \tilde{p}_b(\underline{v}_b))$. If all types of both players are sure to trade, however, then the Chatterjee-Samuelson analysis breaks down, and a continuum of equilibria exist in which all types of both parties

name any price in the interval $[\bar{v}_s, \underline{v}_b]$. We deal with this case below.

When no seller-type is sure to trade, calculation shows that the buyer's interim payoff is:

$$(2)\quad U_b(v_b;[\underline{v}_s, \bar{v}_s], [\underline{v}_b, \bar{v}_b]) = \begin{cases} 0 & \text{if } v_b \leq \underline{\beta}, \\ \dfrac{(v_b - \underline{\beta})^2}{2(\bar{v}_s - \underline{v}_s)} & \text{if } \underline{\beta} < v_b \leq \bar{\beta}, \\ v_b - \bar{\beta} + \dfrac{\bar{v}_s - \underline{v}_s}{2} & \text{if } v_b > \bar{\beta}, \end{cases}$$

where $\Delta = (\bar{v}_b - \underline{v}_s)/4$, $\underline{\beta} = \underline{v}_s + \Delta$, and $\bar{\beta} = \bar{v}_s + \Delta$. (In this notation, no seller-type is sure to trade when $\underline{v}_b < \underline{\beta}$.) The three cases in (2) correspond to the cases in which the buyer, given v_b and the supports of the players' types, is sure not to trade, might trade, or is sure to trade, respectively.

When some but not all seller-types are sure to trade (i.e., $\underline{\beta} < \underline{v}_b < \bar{\beta}$), an interval of seller-types trade with the lowest buyer-type. The interim payoff to $\underline{v}_b$ is then

$$(3)\quad \underline{U}_b \equiv U_b(\underline{v}_b;[\underline{v}_s, \bar{v}_s], [\underline{v}_b, \bar{v}_b]) = \frac{(\underline{v}_b - \underline{v}_s - \Delta)^2}{3(\bar{v}_s - \underline{v}_s)},$$

and the interim payoff for other buyer types is

$$(4)\quad U_b(v_b; [\underline{v}_s, \bar{v}_s], [\bar{v}_b, \bar{v}_b]) = \begin{cases} \dfrac{(v_b - \underline{v}_s - \Delta)^2}{2(\bar{v}_s - \underline{v}_s)} - \dfrac{1}{2}\underline{U}_b & \text{if } v_b \leq \bar{\beta}, \\ v_b - \bar{\beta} - \dfrac{1}{2}\underline{U}_b + \dfrac{\bar{v}_s - \underline{v}_s}{2} & \text{if } v_b > \bar{\beta}. \end{cases}$$

Finally, when all seller-types are sure to trade ($\bar{\beta} < \underline{v}_b$), then all buyer -types also are sure to trade, and the Chatterjee-Samuelson analysis breaks

down: the bids given by (1) are irrelevant, and the strategies $p_b(v_b) = \tilde{p}_s(\bar{v}_s)$ and $p_s(v_s) = \tilde{p}_b(\underline{v}_b)$ are <u>not</u> an equilibrium. Without proposing a general theory for this problem, we note that the subgame $\{v_s \varepsilon [0,1-y],\ v_b \varepsilon [y,1]\}$ is symmetric about $\frac{1}{2}$ and so (when $y>\frac{1}{2}$) it is natural to assume that trade will occur with certainty at a price of $\frac{1}{2}$. Then a buyer-type $v_b > \frac{1}{2}$ gets a payoff $(v_b - \frac{1}{2})$.

As described in the text, an equilibrium value of y must satisfy

(5) $(1-y)U_b(y;[0,1-y],\ [y,1] + y\ U_b(y;[1-y,1],\ [y,1])$

$= (1-y)U_b(y;[0,1-y],\ [0,y]),$

since the left-hand side represents y's expected payoff if he says "keen" and the right-hand side represents his payoff if he says "not keen".

Now the first term in (5) is strictly less than the right-hand side, because the only difference is that the seller is more optimistic about the buyer's type. Therefore the second term is strictly positive, so the buyer of type y trades at least sometimes in that subgame ($y > \underline{\beta} = 1- \frac{3}{4}y$, or $y>\frac{4}{7}$), and some seller-types trade for sure. On the other hand, since in this subgame $\bar{\beta} > 1$, not all seller-types trade for sure. Thus (3) applies in the second term of (5).

In the third term, which involves the subgame $\{v_s \varepsilon [0,1-y],\ v_b \varepsilon [0,y]\}$, the critical type $\bar{\beta} = 1- \frac{3}{4}y$, so y trades for sure in that subgame, since $y>\frac{4}{7}$ implies $y>\bar{\beta}$. But $\underline{v}_b = 0 < \underline{\beta} = \frac{1}{4}y$, so no seller-type is sure to trade and the bottom case of (2) applies.

Finally, in the first term, involving the subgame $\{v_s \varepsilon [0,1-y],\ v_b \varepsilon [y,1]\}$ (in which both players are keen), $\bar{\beta} = \frac{5}{4} - y$. So if $y> \frac{5}{4} - y$, or $y>\frac{5}{8}$, then y trades for sure when both agents are keen. This means that all types of both agents trade for sure, and the Chatterjee-Samuelson analysis breaks down, so we impose trade with certainty at price $\frac{1}{2}$. Substituting this and

the other formulae into (5) yields

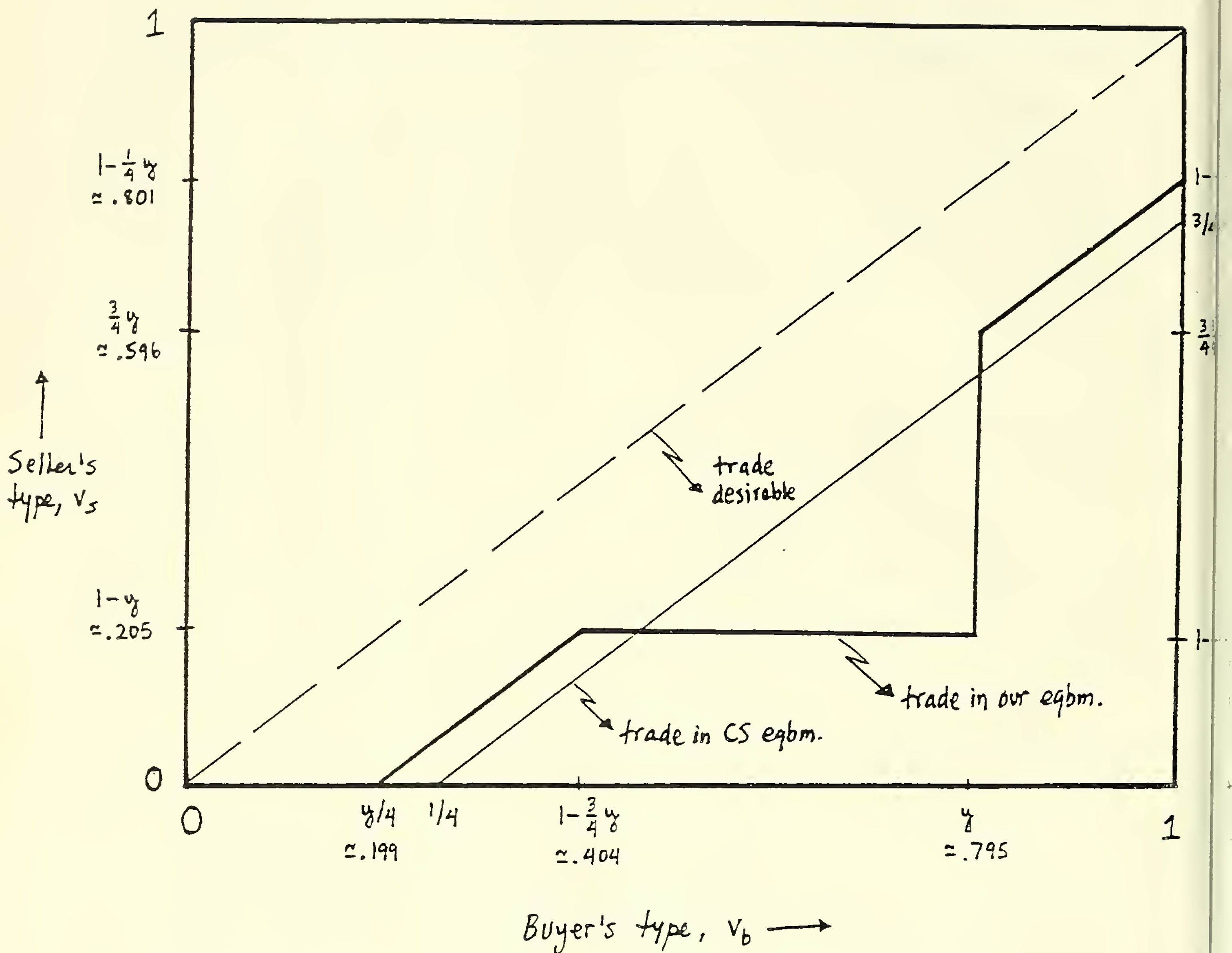
$$(1-y)(y-\frac{1}{2}) + \frac{1}{3}y(\frac{7}{4}y-1)^2 = (1-y)(\frac{5}{4}y - \frac{1}{2}),$$

which has solutions

$$y = \{22 \pm 12\sqrt{2}\}/49$$

$$\approx .103 \text{ or } .795$$

Since the analysis of the second term proves that $y>\frac{4}{7}$, the solution is $y\approx.795$, which exceeds $\frac{5}{8}$, confirming that both parties trade for sure when both are keen.



1
$1-\frac{1}{4}y$
$\simeq .801$
$\frac{3}{4}y$
$\simeq .596$
Seller's type, v_s
$1-y$
$\simeq .205$
0
trade desirable
trade in our eqbm.
trade in CS eqbm.
0
$y/4$
$\simeq .199$
$1/4$
$1-\frac{3}{4}y$
$\simeq .404$
y
$\simeq .795$
1
Buyer's type, v_b

FIGURE 1

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